Math 313A **Homework 3**

Due Wednesday 3/25/09

1. Show that the function $f(z) = e^z$ is entire and use $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ to find $f'(z)$.

2. Use the Cauchy-Riemann equations to show that the following functions are nowhere differentiable.

   (a) $f(z) = \overline{z}$

   (b) $f(z) = \text{Re}\ (z)$

   (c) $f(z) = 2\text{Im}\ (z) - i\text{Re}(z)$ (or if $z = x + yi$ then $f(z) = 2y - ix$)

3. Verify that each function $u(x, y)$ is harmonic and find a harmonic conjugate of u. Express $f(z) = u + iv$ as an analytic function of z.

   (a) $u(x, y) = y$

   (b) $u(x, y) = xy - x + y$

   (c) $u(x, y) = e^x \sin(y)$

4. Prove that if $f(z)$ is analytic in a domain D and if $f'(z) = 0$ everywhere in D, then $f(z)$ is constant in D.