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Find the supremum of each set.

$$S = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}$$

$$S = \{x \in \mathbb{R} : 0 \leq x < 1\}$$

$$S = \left\{ (-1)^n \left( 2 + \frac{1}{n} \right) : n \in \mathbb{N} \right\}$$

$$S = \{\tan^{-1}(x) : x \in \mathbb{R}\}$$

$$S = \{(-1)^n : n \in \mathbb{N}\}$$

1. For each of the sets above, if  $M = \sup(S)$  is it possible to find an element of the set sitting in the interval  $(M - \frac{1}{2}, M]$ ? If so, describe the elements of each set satisfying this condition. If not, explain why.
2. For each of the sets above, if  $M = \sup(S)$  and  $\varepsilon > 0$  is any positive number, is it possible to find an element of the set sitting in the interval  $(M - \varepsilon, M]$ ? If so, describe the elements of each set satisfying this condition. If not, explain why.
3. Now repeat the previous questions with any upper bound  $u \neq \sup(S)$  for each set above. That is, can you find an element of the set sitting in the interval  $(u - \frac{1}{2}, u]$ ? If  $\varepsilon$  is any positive number can you find an element of the set in the interval  $(u - \varepsilon, u]$ ? Explain.

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Use this to formulate a new definition of supremum:

If  $S$  is a nonempty subset of real numbers, then  $M = \sup(S)$  if and only if...