Math 407A Homework 3

Fall 2007 Due Wednesday 9/26/07

(1) Show that

$$\sup\{x \in \mathbb{Q} : x > 0, x^2 < 2\} = \sqrt{2}.$$

(2) Let $S \subset \mathbb{R}$ be nonempty. Define

$$-S = \{x : -x \in S\}.$$

Prove one of the following.

$$\sup(-S) = -\inf(S)$$

$$\inf(-S) = -\sup(S).$$

(3) Let $A, B \subset \mathbb{R}$ be nonempty. Define

$$A + B = \{x + y : x \in A, y \in B\}$$

$$A - B = \{x - y : x \in A, y \in B\}.$$

(a) Prove one of the following.

$$\sup(A + B) = \sup(A) + \sup(B)$$

$$\sup(A - B) = \sup(A) - \inf(B).$$

(b) What are the analogous results for $\inf(A + B)$ and $\inf(A - B)$?

(4) Given nonempty subsets A and B of *positive* real numbers, define

$$AB = \{xy : x \in A, y \in B\}$$

$$\frac{1}{A} = \left\{\frac{1}{x} : x \in A\right\}.$$

Prove one of the following.

(a) $\sup(AB) = \sup(A)\sup(B)$.

(b) If $\inf(A) > 0$, then

$$\sup\left(\frac{1}{A}\right) = \frac{1}{\inf(A)}.$$

(c) If $\inf(A) = 0$, then

$$\sup\left(\frac{1}{A}\right) = +\infty.$$

(5) Let A and B be nonempty subsets of $\mathbb{R}$. Prove one of the following.

$$\sup(A \cup B) = \max\{\sup(A), \sup(B)\}$$

$$\inf(A \cup B) = \min\{\inf(A), \inf(B)\}$$