
I. Fill in the following blanks.

1. The Completeness Axiom says that any nonempty subset of real numbers that is _____ has a _____.
2. A set S is called inductive if $n \in S$ implies _____.
3. Let S be a nonempty subset of real numbers that is bounded above. A real number M_0 is the supremum of S if and only if
 - (a) _____
 - (b) If M is any other upper bound for S , then _____.
4. Let S be a nonempty subset of real numbers that is bounded above. A real number M_0 is the supremum of S if and only if
 - (a) _____
 - (b) For any $\varepsilon > 0$ _____.
5. Let $S, T \subseteq \mathbb{R}$ be nonempty with $S \subseteq T$. Then
$$\inf(\underline{\quad}) \leq \inf(\underline{\quad}) \leq \sup(\underline{\quad}) \leq \sup(\underline{\quad}).$$

6. Find the maximum, minimum, supremum, and infimum if they exist for each of the following sets.

| S | $\max(S)$ | $\min(S)$ | $\sup(S)$ | $\inf(S)$ |
|---|-----------|-----------|-----------|-----------|
| $\left\{ \frac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime} \right\}$ | | | | |
| $\bigcup_{n=1}^{\infty} [2n, 2n+1]$ | | | | |
| $\left\{ (-1)^n \left(2 + \frac{1}{3^n} \right) : n \in \mathbb{N} \right\}$ | | | | |
| $\bigcap_{n=1}^{\infty} \left[1 - \frac{1}{n}, 1 + \frac{1}{n} \right]$ | | | | |
| $\left\{ \cos \left(\frac{n\pi}{3} \right) : n \in \mathbb{N} \right\}$ | | | | |

II. Determine whether the following are True or False. If the statement is true, prove it. If it is false, give a counter-example.

1. If $S, T \subseteq \mathbb{R}$ are nonempty subsets such that $s \leq t$ for all $s \in S$ and $t \in T$, and $\sup(S) = \inf(T)$, then $S \cap T = \emptyset$.
2. If $S, T \subseteq \mathbb{R}$ are nonempty subsets such that $s \leq t$ for all $s \in S$ and $t \in T$, and $\sup(S) = \inf(T)$, then $S \cap T \neq \emptyset$.
3. If S and T are nonempty bounded subsets of \mathbb{R} , then $\sup(S \cup T) = \sup(S) + \sup(T)$.
4. If S and T are nonempty bounded subsets of \mathbb{R} , then $\sup(S \cap T) = \sup(S) - \sup(T)$.
5. If S is a nonempty bounded subset of \mathbb{R} , then $\max(S) \in S$.
6. If S is a nonempty bounded subset of \mathbb{R} , then $\sup(S) \in S$.
7. For $a, b \in \mathbb{R}$, $|b| < a$ if and only if $-a < b < a$.