

The probability of Type I error is α . That is, the probability that we find something in observation (sampling) that is different from what we expect under the conditions of the hypothesis (i.e. if we assume the hypothesis is true).

The *test of significance* tells us how significant the difference between the observed value and the hypothesized value have to be in order to reject the null hypothesis. The maximum probability with which we would be willing to risk a Type I error is called the *level of significance* of the test. This is specified before sampling. Usually the level of significance is set at 0.01 (1%) or 0.05 (5%).

If a 5% level of significance is chosen, then 5% of the time (i.e. 5 chances in 100) we would reject the null hypothesis when it should be accepted. So whenever the hypothesis is true we are 95% confident we would make the right decision. We say the hypothesis has been rejected at a 0.05 level of significance, which means that we could be wrong (i.e. Type I error) with probability 0.05.

If the test statistic, X , we calculate has a normal distribution then the variable

$$Z = \frac{X - \mu_X}{\sigma_X}$$

has a standard normal distribution. The level of significance gives us z -values (z -scores) outside of which we should reject the null hypothesis.

If the test requires we look at extreme values of X (i.e. outside the z -scores on both sides of the mean), then it is a *two-sided test*. If we are interested in extreme values on one side of the mean, then it is a *one-sided test*.

Example 1. null hypothesis: $H_0 : \mu = 12$ (sample size $n = 36$ from population with known standard deviation $\sigma = 3$)

alternative hypothesis: $H_1 : \mu \neq 12$

test of significance: reject if $\bar{x} < 10.71$ or $\bar{x} > 13.29$ (two-sided test)

level of significance: $\alpha = 0.01$

z -scores: -2.58 and 2.58

Example 2. null hypothesis: $H_0 : \mu = 12$ (sample size $n = 36$ from population with known standard deviation $\sigma = 3$)

alternative hypothesis: $H_1 : \mu > 12$

test of significance: reject if $\bar{x} > 13.165$ (one-sided test)

level of significance: $\alpha = 0.01$

z -score: 2.33

The following tables give the z -scores from the standard normal distribution for common confidence levels and significance levels.

Confidence Level	99.73%	99%	98%	96%	95.45%	95%	90%	80%	68.27%	50%
z -score	3.00	2.58	2.33	2.05	2.00	1.96	1.645	1.28	1.00	0.6745

Level of Significance α	0.10	0.05	0.01	0.005	0.002
z -scores for one-sided tests	-1.28 or 1.28	-1.645 or 1.645	-2.33 or 2.33	-2.58 or 2.58	-2.88 or 2.88
z -scores for two-sided tests	-1.645 and 1.645	-1.96 and 1.96	-2.58 and 2.58	-2.81 and 2.81	-3.08 and 3.08