
An *experiment* is any process of observation or measurement. The *sample space* is the set of all possible *outcomes* of the experiment.

Example 1. *Tossing a balanced coin 3 times yields 8 possible outcomes.*

A *random variable*, X, is a real-valued function defined on a sample space and x is a value of the random variable X. It assigns a real number to each outcome.

Example 2. *If X gives the number of heads in 3 tosses of a coin, then the possible values are $x = 0, 1, 2, 3$.*

A random variable that takes on a finite or countably infinite number of values is a *discrete* random variable. A random variable that can take on uncountably many values is a *continuous* random variable.

If X is a discrete random variable, then its *probability distribution* is a function $f(x)$ that gives the probability of each r.v. value. That is, $P(X = x) = f(x)$.

If X is a continuous random variable, then its *probability density* is a function $f(x)$ that gives the probability of a range of r.v. values. That is,

$$P(a \leq X \leq b) = \int_a^b f(x)dx.$$

The *expected value* or *expectation* of a random variable X is a long-term average. The average is weighted by the probability of each r.v. value. The expectation of X is denoted by $E(X)$.

If X is a discrete random variable with probability distribution $f(x)$, then

$$E(X) = \sum_x xf(x).$$

If X is a continuous random variable with density function $f(x)$, then

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx.$$

Example 3. *How many heads should we expect in 3 consecutive tosses of a fair coin?*

Example 4. *If a coin is weighted so that the probability of heads is 0.76, how many heads should we expect to get in 3 consecutive tosses?*

Example 5. *Suppose the time (in minutes) it takes for a laser fired at the moon to be reflected back to its source on earth has density function*

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & otherwise. \end{cases}$$

How long should we expect the laser to take to reach the moon and be reflected back to earth?