**Math 105**

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1. Consider the following table of values.

$$\begin{array}{cccccc} 0 & 1 & 2 & 3 & \dots & 9 \\ 9 & 0 & 1 & 2 & \dots & 8 \\ 8 & 9 & 0 & 1 & \dots & 7 \\ \vdots & & & & & \vdots \\ 1 & 2 & 3 & 4 & \dots & 0 \end{array}$$

   You will choose 10 entries from the table from different rows and different columns. That is, pick only one number from each row and column.

   (a) Is it possible to choose the same number 10 times?

   (b) Is it possible to have the same number repeated 9 times? 8 times?

   (c) Is it possible for each number to be chosen only once? That is, there are no repeated numbers in your list.

   (d) What is the possible range for the sum of your digits?

2. Now consider the following table.

| | | | | |
|---|---|---|---|---|
| 19 | 8 | 11 | 25 | 7 |
| 12 | 1 | 4 | 18 | 0 |
| 16 | 5 | 8 | 22 | 4 |
| 21 | 10 | 13 | 27 | 9 |
| 14 | 3 | 6 | 20 | 2 |

   Again you will choose 10 entries from the table from different rows and columns.

(a) Do you notice any pattern in your choices? *(You will probably have to repeat the selection several times before you notice the pattern.)*

(b) Carefully study the entries in the table to determine how it was built. *Hint: You could look at the first row as:*

| 7+12 | 7+1 | 7+4 | 7+18 | 7+0 |
|---|---|---|---|---|
| 12 | 1 | 4 | 18 | 0 |
| 16 | 5 | 8 | 22 | 4 |
| 21 | 10 | 13 | 27 | 9 |
| 14 | 3 | 6 | 20 | 2 |

(c) Build your own table of values that will yield a similar result.

(d) What pattern do you notice in the numbers used to build your table?

3. The numbers $1, 2, \ldots, n^2$ are arranged in an $n \times n$ table as follows.

| | | | | |
|---|---|---|---|---|
| $1$ | $2$ | $3$ | $\ldots$ | $n$ |
| $n+1$ | $n+2$ | $n+3$ | $\ldots$ | $2n$ |
| $2n+1$ | $2n+2$ | $2n+3$ | $\ldots$ | $3n$ |
| $\vdots$ | | | | $\vdots$ |
| $n^2-n+1$ | $n^2-n+2$ | $n^2-n+3$ | $\ldots$ | $n^2$ |

Pick $n$ numbers from the table from different rows and columns. That is, one number from each row and column. Find the sum of these numbers.