	Expression	$\lim_{x\to u} f(x)$	$\lim_{x\to u} g(x)$	Indeterminate Form
(1)	$\dfrac{f(x)}{g(x)}$	0	0	$\frac{0}{0}$
(2)	$\dfrac{f(x)}{g(x)}$	∞	∞	$\frac{\infty}{\infty}$
(3)	$f(x) \cdot g(x)$	0	∞	$0 \cdot \infty$
(4)	$f(x) - g(x)$	∞	∞	$\infty - \infty$
(5)	$[f(x)]^{g(x)}$	0	0	0^0
(6)	$[f(x)]^{g(x)}$	∞	0	∞^0
(7)	$[f(x)]^{g(x)}$	1	∞	1^∞

Here the limit can be any of: $\lim_{x\to a}$ $\lim_{x\to a^-}$ $\lim_{x\to a^+}$ $\lim_{x\to -\infty}$ $\lim_{x\to +\infty}$.

L'Hôpital's Rule

(For expressions **(1)** and **(2)**)

If $\dfrac{f(x)}{g(x)}$ has the indeterminate form $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$ at u and $\lim_{x\to u} \dfrac{f'(x)}{g'(x)}$ exists
(i.e. this limit is a finite number or $-\infty$ or ∞)

then

$$\boxed{\lim_{x\to u} \frac{f(x)}{g(x)} = \lim_{x\to u} \frac{f'(x)}{g'(x)}}$$

For expression **(3)**

If $f(x) \cdot g(x)$ has the indeterminate form $\boxed{0 \cdot \infty}$ at u, then rewrite:

$$f(x) \cdot g(x) = \frac{f(x)}{1/g(x)}$$, which has the indeterminate form $\boxed{\frac{0}{0}}$ at u

or

$$f(x) \cdot g(x) = \frac{g(x)}{1/f(x)}$$, which has the indeterminate form $\boxed{\frac{\infty}{\infty}}$ at u

and then apply L'Hôpital's Rule.

For Expression **(4)**

If $f(x) - g(x)$ has the indeterminate form $\boxed{\infty - \infty}$ at u,

then use algebraic manipulation to convert $f(x) - g(x)$

into a form of the type $\boxed{\frac{0}{0}}$ or $\boxed{\frac{\infty}{\infty}}$

and then apply L'Hôpital's Rule.

For expression **(5)**

If $[f(x)]^{g(x)}$ has the indeterminate form $\boxed{0^0}$ at u, then follow these steps:

Let

$$y = [f(x)]^{g(x)}$$

So

$$\ln y = \ln\Big([\ f(x)\ ]^{g(x)} \Big)$$

Next, simplify

$$\ln y = g(x)\ln(f(x))$$

Note that $\ln y = g(x)\ln(f(x))$ has the indeterminate form $\boxed{0 \cdot -\infty}$ as $x \to u$.

Using an appropriate above method (i.e. **(3)**), evaluate

$$\lim_{x \to u} \ln y = L\ .$$

Conclude

$$\lim_{x \to u} \ln [f(x)]^{g(x)} = L \qquad \Longrightarrow \qquad \lim_{x \to u} [f(x)]^{g(x)} = e^L\ .$$

For expressions **(6)** and **(7)**

If $[f(x)]^{g(x)}$ has the indeterminate form $\boxed{\infty^0}$ or $\boxed{1^\infty}$, then proceed similarly as in **(5)** .

Note that $\ln y = g(x)\ln(f(x))$ will have the indeterminate form

- **(6)** $\boxed{0 \cdot \infty}$ as $x \to u$
- **(7)** $\boxed{\infty \cdot 0}$ as $x \to u$.
